Que:- What is polarizability? Deduce clausius - Mossotti relation between polarizability and dielectric const.

Dielectrics are basically electric insulators which donot contain any free charge carriers for conduction. They contain positive and negative charges which are bound together ~~and~~ which affected by the applied electric fields.

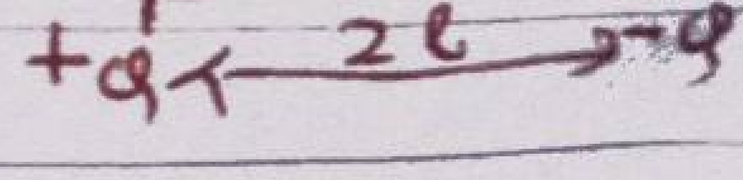
Polarization and succeptibility


When a dielectric is placed in an external electric field Eo its molecules become electric dipoles oriented in the direction of the field. Hence the dielectric acquires a net dipole moment and its molecules are called polarized.

In the case of molecules the dipole moment is proportional to the field causing polarization, The dipole moment of a molecule per unit polarising field is called the **Polarizability** of the molecule.

Polarization P is the dipole moment per unit volume by the external field.

$$P = p/V \qquad p = 2ql$$




The magnitude of the resultant field is less than

The applied field i.e. $E < E_0$

$E = E_0 + E_P \rightarrow$ polarization field as it tends to oppose the applied field $E_0$ within the material.

Polarization $P = q - q'$ $\qquad \epsilon_r = E/E_0$

$$= \frac{1}{4\pi}(E - E_0)$$

$$= \frac{1}{4\pi}(\epsilon_r E_0 - E_0)$$

$$\boxed{P = \frac{E_0}{4\pi}(\epsilon_r - 1)}$$

Dielectric constant

Electric field

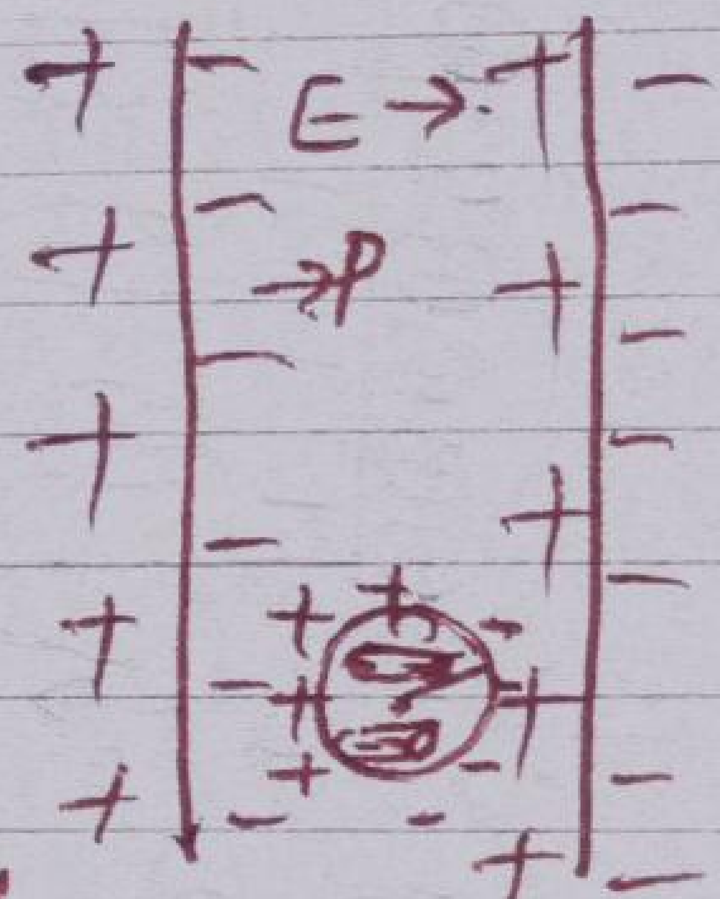
ratio of $P/E = \chi$ susceptibility

in free space $P = 0$, $E = 0$

$\chi = 0$

## The local field →

The electric field acting at the site of an atom or molecule is different from the macroscopic electric field E and is called the local field. This field is responsible for polarization of each atom or molecule of a solid



$$E_{loc} = E_0 + E_p + \frac{P}{3\epsilon_0} = E + P/3\epsilon_0$$

$$= E_0 + E_P + E_S + E_m$$

~~Dielectric constant and Polarizability~~

Here $E_0$ = The primary electric field due to the charge on the plates

$E_P$ = The field due to the polarization charges at the plate-dielectric interface.

$E_s$ = The field due to the charges induced at the spherical surface

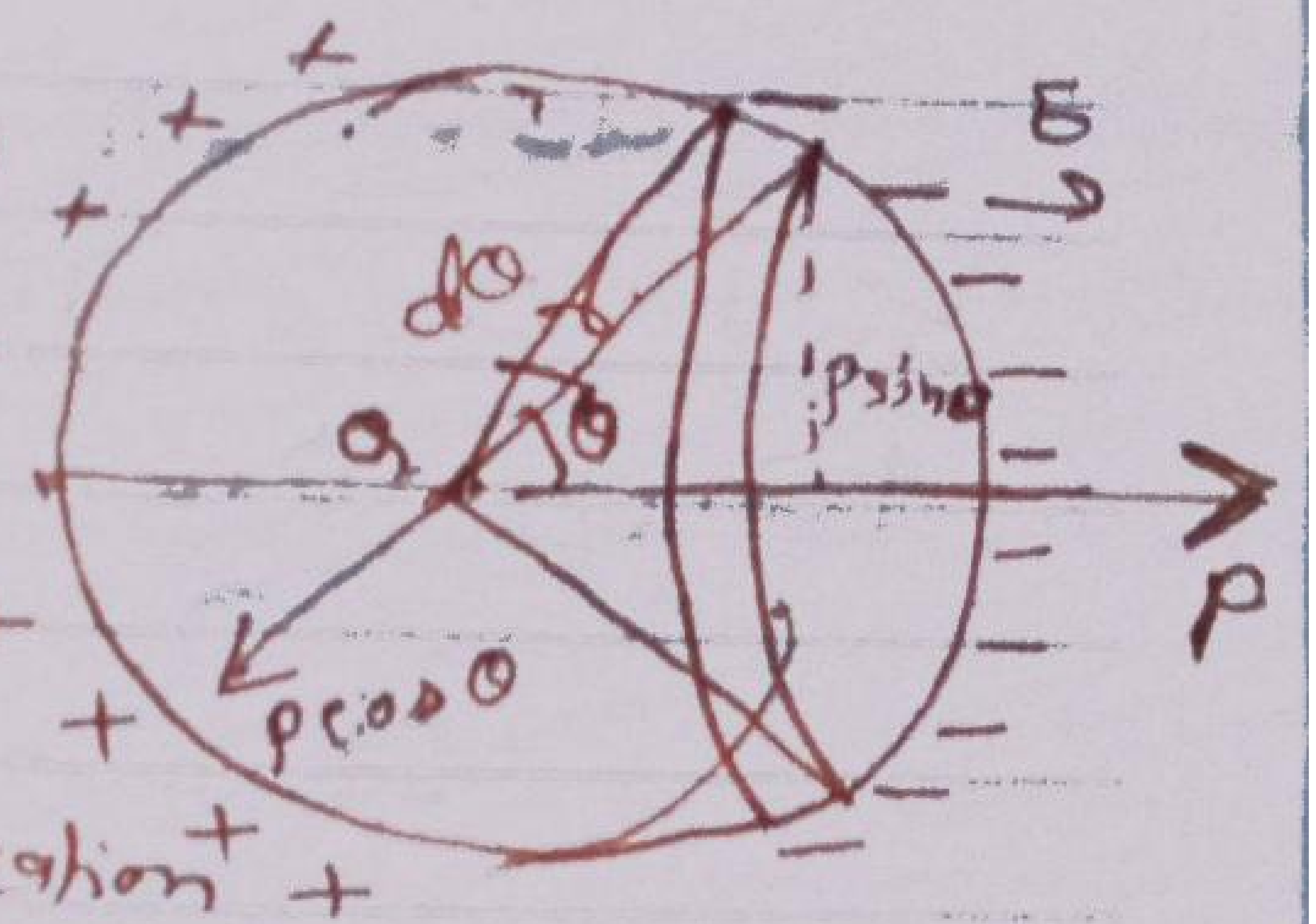
$E_m$ = Due to all the dipoles of the atoms inside the spherical region.

$E_0 + E_p = E$ = The macroscopic electric field inside the dielectric. Hence

$E_{loc} = E + E_s + E_m$

$E_m = 0$ for cubic crystals, Dipoles inside the spherical surface are randomly distributed in position.

$$\boxed{E_{loc} = E + E_s}$$




$E_s \rightarrow$ The charge ~~element~~ density ~~of~~ in a surface element $ds$ of the sphere is equal to the normal component of the polarization times the surface element ie ($-p\cos\theta \, ds$). According to the coulomb's law this charge element produces a force $dF_r$

$$dF_r = \frac{q_1 q_2}{r^2} = -\frac{q\, p\cos\theta \, dA}{r^2}$$

acting on a test charge q at the centre of the sphere in the dirn.

DECEMBER 2015 / JANUARY 2016

Hence, the field $dE_s$ at the centre due to this charge element is

$$dE_s = \frac{dF_r}{q} = -\frac{P\cos\theta\, ds}{r^2}$$

Thus only the component of $dE_s$ along the direction of P will contribute to the integral of above equation over the entire surface thus $dE_s \cdot \cos\theta$.

$$E_s = -\int \frac{P\cos\theta\, ds}{r^2} \cdot \cos\theta$$

$$E_s = -\int \frac{P\cos^2\theta\, ds}{r^2}$$

$ds = 2\pi r \cdot \sin\theta\, r d\theta$ (Ring surface)

$$ds = 2\pi r^2 \sin\theta\, d\theta \quad \leftarrow 23$$

and the limits of integration with respect to $\theta$ are from 0 to $\pi$. Thus

$$E_s = -\int_0^{\pi} \frac{P\cos^2\theta}{\cancel{r^2}} \cdot 2\pi \cancel{r^2} \sin\theta\, d\theta$$

$$= -2\pi P \int_0^{\pi} \cos^2\theta \sin\theta\, d\theta$$

let

$$z = \cos\theta \qquad dz = -\sin\theta\, d\theta$$

$$E_s = +2\pi P \int_1^{1} z^2 dz = -2\pi P \cdot \frac{2z}{3}$$

$$= -\frac{4\pi P}{3} \text{ Case } \theta = 0 \text{ Case}$$

$$\boxed{E_s = \frac{4\pi P}{3}}$$

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$$\boxed{\begin{aligned} E_{loc} &= E + \underline{4\pi P} \\ D &= E + 4\pi P \end{aligned}}$$

Lorentz field
Lorentz relation.

# The clausius- MOSSOTTI Relation →

The dipole moment of single atom p is proportional to the local field

$$p = \alpha E_{loc}$$

where $\alpha$ is const called electrical polarizability of the atom.

The total polarization of an insulator containing N types of atoms is

$$P = \sum_{j=1}^{N} n_i \alpha_i E_{loc} = E_{loc} \sum_{j=1}^{N} n_i \alpha_i$$

where $n_i$ = The number of i atoms per unit volume

$\alpha_i$ = polarizabilities of i atoms

$$P = \left(E + \frac{4\pi P}{3}\right) \sum_{j=1}^{N} n_i \alpha_i$$

$$\cancel{= E\left(1 + \frac{4\pi}{3}\frac{P}{E}\right) \sum_{j=1}^{N} n_i \alpha_i}$$

NOTES

$$\frac{P}{} - \frac{4\pi P}{3} \sum_{j=1}^{n} n_i \alpha_i = E \sum_{j=1}^{n} n_i \alpha_i$$

$$P\left(1 - \frac{4\pi}{3} \sum_{j=1}^{n} n_i \alpha_i\right) = E \sum_{j=1}^{n} n_i \alpha_i$$

$$P/E = \left(\sum_{j=1}^{n} n_i \alpha_i\right) \Big/ 1 - \frac{4\pi}{3} \sum_{j=1}^{n} n_i \alpha_i$$

$$P = (\epsilon - 1)\frac{E}{4\pi}$$

$$\chi = \frac{P}{E} = \frac{(\epsilon-1)}{4\pi} \rightarrow \text{Electric susceptibility}$$

So $$\frac{\epsilon - 1}{4\pi} = \frac{\sum_j n_j \alpha_j}{1 - \left(\frac{4\pi}{3}\right)\sum_j n_j \alpha_j}$$

$$(\epsilon - 1)\left\{1 - \frac{4\pi}{3}\sum_j n_j \alpha_j\right) = \cancel{4\pi} \sum_{j=}^{n} n_j \alpha_j$$

$$(\epsilon - 1) - (\epsilon - 1)\frac{4\pi}{3}\sum_{j=1}^{n} n_j \alpha_j = \sum_{j=1}^{n} n_j \alpha_j$$

$$(\epsilon - 1) = \sum_{j=1}^{n} n_j \alpha_j + (\epsilon - 1)\frac{4\pi}{3}\sum_{j=1}^{n} n_j \alpha_j$$

$$= 4\pi\sum_{j=1}^{n} n_j \alpha_j \left\{1 + \frac{(\epsilon - 1)}{3}\cancel{\frac{\ }{3}}\right\}$$

$$= 4\pi\sum_{j=1}^{n} n_j \alpha_j \left\{\frac{3 + \epsilon - 1}{3}\right\}$$

$$= \frac{4\pi}{3}\sum_{j=1}^{n} n_j \alpha_j (\epsilon + 2)$$

31 SUNDAY

NOTES

$$\boxed{\frac{(\epsilon - 1)}{(\epsilon + 2)} = \frac{4\pi}{3}\sum_{j=1}^{n} n_j \alpha_j}$$

This is called Claussius - Masotti equation.

$$\sum_j n_j \alpha_j = n\alpha$$

$N_A$ = Avogadro number $\quad n = \frac{\rho N_A}{M} \quad \rho$ = density

$M$ = Molecular weight

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3} \frac{\rho N_A}{M} \alpha$$

$\frac{(\epsilon - 1)}{(\epsilon + 2)}$ = Atomic polarizability

$$\boxed{\frac{M}{\rho} \frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3} \cdot N_A \alpha}$$

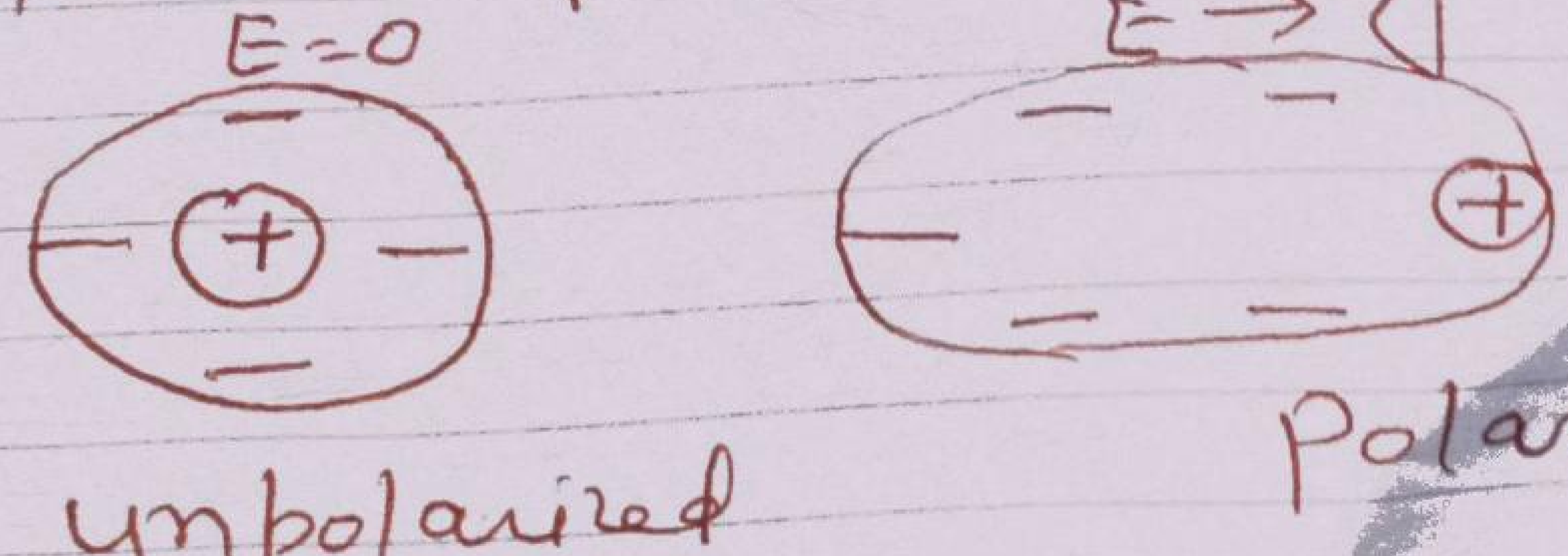
Claussius - MASOTTI Equation

**Sources of Polarizability →**

The net polarizability of a dielectric material results mainly from the following three types contributions.

1. Electronic polarizability
2. Ionic polarizability
3. Dipolar or orientational ~~polariza~~ polarizability

Electronic Polarizability →




unpolarized atom

Polarized atom

NOTES

The electronic polarizability arises due to the displacement of the electron cloud of an atom relative to its nucleus in the presence of an applied electric field. The polarization as well as the dielectric constant of a material at optical frequencies mainly from the electronic polarizability.

At optical frequencies

$$\frac{n^2-1}{n^2+2} = \frac{1}{3\epsilon_0} \sum_j n_j \alpha_i \text{ (Electronic)}$$

where $\epsilon_r$ has been replaced by $n^2$

$n =$ Refractive index